CS2021 Assignment 1 Answers

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For each question in the assignment, there are more than one solution. The solutions described below are just for reference. If you have any question regarding your own solution, please contact Professor John Sum for validation.

Contents

1 Question 1

1.1 Solution 1

Initially, label the balls with numbers $1, 2, \cdots, 9$.

```
Step 1: Weight B1 and B2. GOTO Step 2.
Step 2: IF B1 < B2, B1 is abnormal. STOP! ELSE GOTO Step 3.
Step 3: IF B1 > B2, B2 is abnormal. STOP! ELSE GOTO Step 4.
Step 4: SET N = 3. GOTO Step 5.
STEP 5: 5.1: Weight B1 and BN.
        5.2: IF B1 > BN, BN is abnormal. STOP! ELSE GOTO Step 5.3.
        5.3: N = N+1. GOTO Step 5.1.
```

For the above algorithm, the number of WEIGHT for finding the abnormal ball is no more than 7. It happens if the abnormal ball is B_9.

1.2 Solution 2

Initially, arbitrarily partition the balls in three groups, say Group A, B and C. Each group has three balls. Label the balls A_1, A_2 and A_3 for the balls in Group A. Label the balls B_1, B_2 and B_3 for the balls in Group B. Label the balls C_1, C_2 and C_3 for the balls in Group C.

```
Step 1: Weight (A1,A2,A3) and (B1,B2,B3). GOTO Step 2.
Step 2: IF (A1,A2,A3) < (B1,B2,B3), GOTO Step 5.
Step 3: IF (A1,A2,A3) > (B1,B2,B3), GOTO Step 6.
Step 4: IF (A1,A2,A3) = (B1,B2,B3), GOTO Step 7.
Step 5: 5.1: Weight A1 and A2. GOTO Step 5.2.
        5.2: IF A1 < A2, A1 is abnormal. STOP! ELSE GOTO Step 5.3.
        5.3: IF A1 > A2, A2 is abnormal. STOP!
             ELSE A3 is abnormal. STOP!
Step 6: 6.1: Weight B1 and B2. GOTO Step 6.2.
        6.2: IF B1 < B2, B1 is abnormal. STOP! ELSE GOTO Step 6.3.
        6.3: IF B1 > B2, B2 is abnormal. STOP!
             ELSE B3 is abnormal. STOP!
Step 7: 7.1: Weight C1 and C2. GOTO Step 7.2.
        7.2: IF C1 < C2, C1 is abnormal. STOP! ELSE GOTO Step 7.3.
        7.3: IF C1 > C2, C2 is abnormal. STOP!
             ELSE A3 is abnormal. STOP!
```

For the above algorithm, the number of WEIGHT for finding the abnormal ball is no more than 2.

2 Question 2

2.1 Solution 1

Initially, label the balls with numbers $1, 2, \cdots, 9$.

```
Step 1: Weight B1 and B2. GOTO Step 2.
Step 2: IF B1 < B2, B2 is abnormal. STOP! ELSE GOTO Step 3.
Step 3: IF B1 > B2, B1 is abnormal. STOP! ELSE GOTO Step 4.
Step 4: SET N = 3. GOTO Step 5.
STEP 5: 5.1: Weight B1 and BN.
        5.2: IF B1 < BN, BN is abnormal. STOP! ELSE GOTO Step 5.3.
        5.3: N = N+1. GOTO Step 5.1.
```

For the above algorithm, the number of WEIGHT for finding the abnormal ball is no more than 7. It happens if the abnormal ball is B_9.

2.2 Solution 2

Initially, arbitrarily partition the balls in three groups, say Group A, B and C. Each group has three balls. Label the balls A_1, A_2 and A_3 for the balls in Group A. Label the balls B_1, B_2 and B_3 for the balls in Group B. Label the balls C_1, C_2 and C_3 for the balls in Group C.

```
Step 1: Weight (A1,A2,A3) and (B1,B2,B3). GOTO Step 2.
Step 2: IF (A1,A2,A3) > (B1,B2,B3), GOTO Step 5.
Step 3: IF (A1,A2,A3) < (B1,B2,B3), GOTO Step 6.
Step 4: IF (A1,A2,A3) = (B1,B2,B3), GOTO Step 7.
Step 5: 5.1: Weight A1 and A2. GOTO Step 5.2.
        5.2: IF A1 > A2, A1 is abnormal. STOP! ELSE GOTO Step 5.3.
        5.3: IF A1 < A2, A2 is abnormal. STOP!
             ELSE A3 is abnormal. STOP!
Step 6: 6.1: Weight B1 and B2. GOTO Step 6.2.
        6.2: IF B1 > B2, B1 is abnormal. STOP! ELSE GOTO Step 6.3.
        6.3: IF B1 < B2, B2 is abnormal. STOP!
             ELSE B3 is abnormal. STOP!
Step 7: 7.1: Weight C1 and C2. GOTO Step 7.2.
        7.2: IF C1 > C2, C1 is abnormal. STOP! ELSE GOTO Step 7.3.
        7.3: IF C1 < C2, C2 is abnormal. STOP!
             ELSE A3 is abnormal. STOP!
```

For the above algorithm, the number of WEIGHT for finding the abnormal ball is no more than 2.

3 Question 3

3.1 Solution 1

Initially, label the balls with numbers $1, 2, \cdots, 9$.

```
Step 1: Weight B1 and B2. GOTO Step 2.
Step 2: IF (B1 < B2) or (B1 > B2), GOTO Step 3. ELSE GOTO Step 4.
Step 3: 3.1: Weight B1 and B3.
        3.2: IF B1 = B3, B2 is abnormal.
             ELSE, B1 is abnormal. STOP!
Step 4: SET N = 3. GOTO Step 5.
STEP 5: 5.1: Weight B1 and BN.
        5.2: IF B1 = BN, GOTO Step 5.3. ELSE, BN is abnormal. STOP!
        5.3: N = N+1. GOTO Step 5.1.
```

For the above algorithm, the number of WEIGHT for finding the abnormal ball is no more than 7. It happens if the abnormal ball is B_9.

3.2 Solution 2

Initially, arbitrarily partition the balls in three groups, say Group A, B and C. Each group has three balls. Label the balls A_1, A_2 and A_3 for the balls in Group A. Label the balls B_1, B_2 and B_3 for the balls in Group B. Label the balls C_1, C_2 and C_3 for the balls in Group C.

```
Step 1: 1.1: Weight (A1,A2,A3) and (B1,B2,B3). GOTO Step 1.2.
        1.2: SET (GA > GB) = TRUE, IF (A1,A2,A3) > (B1,B2,B3).
             GOTO Step 2. ELSE GOTO Step 1.3.
        1.3: SET (GA < GB) = TRUE, IF (A1,A2,A3) < (B1,B2,B3).
             GOTO Step 2. ELSE SET (GA = GB) = TRUE. GOTO Step 2.
Step 2: 2.1: Weight (A1,A2,A3) and (C1,C2,C3). GOTO Step 2.2.
        2.2: SET (GA > GC) = TRUE, IF (A1,A2,A3) > (C1,C2,C3).
             GOTO Step 3. ELSE GOTO Step 2.3.
        2.3: SET (GA < GC) = TRUE, IF (A1,A2,A3) < (C1,C2,C3).
             GOTO Step 3. ELSE SET (GA = GC) = TRUE. GOTO Step 3.

[Note: After Step 2, it comes up with six possible outcomes only.]

Step 3: 3.1: IF (GA < GB) and (GA < GC), GOTO Step 4.
             ELSE GOTO Step 3.2.
        3.2: IF (GA > GB) and (GA = GC), GOTO Step 5.
             ELSE GOTO Step 3.3.
        3.3: IF (GA = GB) and (GA > GC), GOTO Step 6.
             ELSE GOTO Step 3.4.
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        3.4: IF (GA > GB) and (GA > GC), GOTO Step 7.
             ELSE GOTO Step 3.5.
        3.5: IF (GA < GB) and (GA = GB), GOTO Step 8.
             ELSE GOTO Step 3.6.
        3.6: IF (GA = GB) and (GA < GC), GOTO Step 9.

[The abnormal ball is lighter.]
Step 4: 4.1: Weight A1 and A2. GOTO Step 4.2.
        4.2: IF A1 < A2, A1 is abnormal. STOP! ELSE GOTO Step 4.3.
        4.3: IF A1 > A2, A2 is abnormal. STOP!
             ELSE A3 is abnormal. STOP!
Step 5: 5.1: Weight B1 and B2. GOTO Step 5.2.
        5.2: IF B1 < B2, B1 is abnormal. STOP! ELSE GOTO Step 5.3.
        5.3: IF B1 > B2, B2 is abnormal. STOP!
             ELSE B3 is abnormal. STOP!
Step 6: 6.1: Weight C1 and C2. GOTO Step 6.2.
        6.2: IF C1 < C2, C1 is abnormal. STOP! ELSE GOTO Step 6.3.
        6.3: IF C1 > C2, C2 is abnormal. STOP!
             ELSE A3 is abnormal. STOP!

[The abnormal ball is heavier.]
Step 7: 7.1: Weight A1 and A2. GOTO Step 7.2.
        7.2: IF A1 > A2, A1 is abnormal. STOP! ELSE GOTO Step 7.3.
        7.3: IF A1 < A2, A2 is abnormal. STOP!
             ELSE A3 is abnormal. STOP!
Step 8: 8.1: Weight B1 and B2. GOTO Step 8.2.
        8.2: IF B1 > B2, B1 is abnormal. STOP! ELSE GOTO Step 8.3.
        8.3: IF B1 < B2, B2 is abnormal. STOP!
             ELSE B3 is abnormal. STOP!
Step 9: 9.1: Weight C1 and C2. GOTO Step 9.2.
        9.2: IF C1 > C2, C1 is abnormal. STOP! ELSE GOTO Step 9.3.
        9.3: IF C1 < C2, C2 is abnormal. STOP!
             ELSE A3 is abnormal. STOP!
```

For the above algorithm, the number of WEIGHT for finding the abnormal ball is 3.

3.3 Solution 3

Weight B_1 to B_N for $N = 2, \cdots, 9$ and record all the results. Check from the following table to find out which ball is abnormal.

In the following table, the notation (i, j) refers to weighting B_i and B_j. The symbol $>$ below (i, j) means that $B_i > B_j$. That is to say, B_i is heavier than B_j.

$(1,2)$	$(1,3)$	$(1,4)$	$(1,5)$	$(1,6)$	$(1,7)$	$(1,8)$	$(1,9)$	Abnormal	L/H
$<$	$<$	$<$	$<$	$<$	$<$	$<$	$<$	B_1	L
$>$	$=$	$=$	$=$	$=$	$=$	$=$	$=$	B_2	L
$=$	$>$	$=$	$=$	$=$	$=$	$=$	$=$	B_3	L
$=$	$=$	$>$	$=$	$=$	$=$	$=$	$=$	B_4	L
$=$	$=$	$=$	$>$	$=$	$=$	$=$	$=$	B_5	L
$=$	$=$	$=$	$=$	$>$	$=$	$=$	$=$	B_6	L
$=$	$=$	$=$	$=$	$=$	$>$	$=$	$=$	B_7	L
$=$	$=$	$=$	$=$	$=$	$=$	$>$	$=$	B_8	L
$=$	$=$	$=$	$=$	$=$	$=$	$=$	$>$	B_9	L
$>$	$>$	$>$	$>$	$>$	$>$	$>$	$>$	B_1	H
$<$	$=$	$=$	$=$	$=$	$=$	$=$	$=$	B_2	H
$=$	$<$	$=$	$=$	$=$	$=$	$=$	$=$	B_3	H
$=$	$=$	$<$	$=$	$=$	$=$	$=$	$=$	B_4	H
$=$	$=$	$=$	$<$	$=$	$=$	$=$	$=$	B_5	H
$=$	$=$	$=$	$=$	$<$	$=$	$=$	$=$	B_6	H
$=$	$=$	$=$	$=$	$=$	$<$	$=$	$=$	B_7	H
$=$	$=$	$=$	$=$	$=$	$=$	$<$	$=$	B_8	H
$=$	$=$	$=$	$=$	$=$	$=$	$=$	$<$	B_9	H

For the above algorithm, the number of WEIGHT is 8. While the above algorithm takes a few more WEIGHT, its idea could easily be extended to solve the problem with 2, 3 or 4 abnormal balls.