Stochastic analysis of mobile agent-based e-shopping

Wenyu Qu*

Graduate School of Information Science,
Japan Advanced Institute of Science and Technology,
1-1 Asahidai, Tatsunokuchi, Ishikawa 923-1292, Japan
E-mail: qu_wenyu@jaist.ac.jp
*Corresponding author

Hong Shen

Graduate School of Information Science,
Japan Advanced Institute of Science and Technology,
1-1 Asahidai, Tatsunokuchi, Ishikawa 923-1292, Japan
E-mail: shen@jaist.ac.jp

John Sum

Department of Computing,
Hong Kong Polytechnic University,
Hung Hom, KLN, Hong Kong
E-mail: cspfsum@comp.polyu.edu.hk

Abstract: In this paper, we consider the problem of mobile agent-based e-shopping. We analyse the probability of success (the probability that an agent can find the destination shop) and the population growth of mobile agents. We found that though a single mobile agent may act randomly, the behaviour of all mobile agents as a whole has some stochastic properties. Our mathematical analysis provides a way to estimate user satisfaction and network performance. Since there is a trade-off between these two factors, managers can achieve maximum benefits by tuning the relevant parameters in our analysis according to the users' requirements and the characteristics of the network.

Keywords: mobile agent; e-shopping; probability of success; population distribution.

Reference to this paper should be made as follows: Qu, W., Shen, H. and Sum, J. (2005) 'Stochastic analysis of mobile agent-based e-shopping', *Int. J. Electronic Business*, Vol. 3, Nos. 3/4, pp.339–355.

Biographical notes: Wenyu Qu received her BS degree and ME degree from Dalian University of Technology, China, in 1994 and 1997, respectively, and is currently a PhD candidate at Japan Advanced Institute of Science and Technology. Her research interests are in the area of algorithms, computing, and networking. Her research now mainly concentrates on analysis of mobile agents. In 2003, she received a best student paper award from HIS'03.

Hong Shen received the BE degree from the Beijing University of Science and Technology of China, the ME degree from the University of Science and Technology of China, the Phlic and PhD degrees from Abo Akademi University, Finland, all in computer science. He is currently a full professor in the Graduate School of Information Science, Japan Advanced Institute of Science and Technology. Previously, he was a Professor at the Griffith University, Australia. Dr. Shen has published more than 170 technical papers on algorithms, parallel and distributed computing, interconnection networks, parallel databases and data mining, multimedia systems, and networking. He has served as an editor of *Parallel and Distributed Computing Practice*, associate editor of *International Journal of Parallel and Distributed Systems and Networks*, editorial board member of *Parallel Algorithms and Applications*, *International Journal of Computer Mathematics*, and *Journal of Supercomputing*, and chaired various international conferences. Dr. Shen is a recipient of the 1991 National Education Commission Science and Technology Progress Award and the 1992 Sinica Academia Natural Sciences Award.

John Sum received his PhD from the Chinese University of Hong Kong in 1998. Between 1998 and 2000, he taught in the Hong Kong Baptist University and the Open University of Hong Kong. Since 2000, he has been with the Hong Kong Polytechnic University as an assistant professor in the Department of Computing. He has a broad research interest in many fields, including web commerce; complex networks including neural networks, P2P and small world network; intelligent agent for game playing: dice games and poker games; and combinatorial auction for e-marketplace. Sum is also an adjunct associate professor in the Laboratory of Internet Software Technologies, Institute of Software, Chinese Academy of Science. Currently, he is a member of the IEEE SMC Society and the IEEE Neural Networks Society.

1 Introduction

The growth of the internet has created a new era of information technology that has changed the lives of millions of people around the world, especially in developed countries. E-shopping is a new kind of business that has come to play an important role in modern life (Barain, 1998–2003; Ahola et al., 2000; Gallaugher, 2002; Feldman, 2000). People can buy and sell things on the internet without ever having to leave home to go out to a shopping mall. Vendors set up e-shops and ultimately construct an online shopping system. Buyers enter relevant net sites, check information about the goods they are interested in, compare quality and prices in different shops, make decisions, and ultimately make their purchase digitally. On one hand, the seller no longer needs to print and mail catalogues, nor maintain an order-taking department, which dramatically lowers costs. On the other hand, the buyer can easily compare catalogues from different many shops, which leads to more choices of products and more convenient price comparison and decision-making. Even though shopping malls have social and traditional aspects that continue to attract buyers, e-shopping now poses a major challenge to brick-and-mortar retail establishments. A survey of existing techniques for agent-mediated e-commerce is provided in He et al. (2003).

Mobile agents, decision-making programmes that are capable of migrating autonomously from node to node in a computer network, are changing the face of e-business and reshaping current business models (Wagner and Turban, 2002). Milojicic (2000) described mobile agents as autonomous, adaptive, reactive, mobile, cooperative, interactive, and delegated. The potential benefits of this technology, including the reduction of network bandwidth consumption and latency, have drawn a great deal of attention in both academia and industry (Brazier et al., 2002; Claessens et al., 2003; Gray et al., 2000; Lange and Oshima, 1999; Li and Lam, 2002; Sum et al., 2003; Tang and Pagurek, 2002; Wang, 1995). The applications of mobile agents, from electronic commerce to distributed computation, have also been studied extensively. Successful examples of mobile agent applications in e-shopping can be found in Menczer et al. (2002), Marques et al. (1999), Kim and Noh (2003) and Kowalczyk et al. (2002). In general, when a mobile agent is encapsulated with a task, it can be dispatched to a destination. After executing and accomplishing its tasks, the summary report of its trip is sent back to the server.

Though a number of mobile agent-mediated e-commerce systems have been designed and implemented in academic institutions and commercial firms, few fully meet the needs of large, complex applications. If mobile agents are to have significant impact in practice, quantitative studies on their behaviours should be explored, to reveal the properties and nature of the mobile agent approach and ultimately guide future research. Although the efficiency of applying mobile agent techniques has been demonstrated and reported in the literature, the mathematical modelling and analysis of mobile agents' behaviours is still in its infancy. Brewington et al. (1999) formulated a method of mobile agent planning, which is analogous to the 'travelling salesman problem' (Garey and Johnson, 1979), to decide the sequence of nodes to visit in a way that will minimise the total execution time until the desired information is found. In Kim and Robertazzi (2000), several statistical models of mobile agents were considered, including dwell time on nodes, average life span, cloning, the inter-reporting process, and the report arrival process.

In order to conserve network resources and achieve a good probability of success, it is desirable to dispatch a small number of mobile agents. In large-scale networks, mobile agents will be generated frequently. If there are too many agents running in the network, they will consume too many network resources, which will affect network performance and ultimately block the entire network; on the other hand, if there are too few agents running in the network, there is no guarantee that the destination shops will be found quickly. Therefore, it is necessary to analyse both the probability of success and the population growth of mobile agents.

In this paper, we concentrate on the problem of how to measure the behaviour of mobile agents during their search. A preliminary version of this work appeared in Qu et al. (2003). Our contributions are summarised as follows: First, we estimate the probability, $P^*(d)$, that k agents can find the destination in d jumps:

$$P^*(d) \leq 1-[(n-1)/(n+\sigma_1-1)]^k,$$

where n is the number of the nodes in the network, and σ_1, σ_n are the largest and smallest number of neighbouring hosts for each node in the network. Second, the population distribution number of mobile agents is presented, $\vec{p}(t) = km\vec{e} + A\vec{p}(t-1)$, where k is the number of agents generated per request, m is the average number of request keyed in a host at any time, $\vec{e} = (1,1,\ldots,1)^T$, and A is a matrix that is derived from the connectivity matrix of the network. The total number of agents running in the network is less than $(n^2\sigma_1 km)/(n+\sigma_1-1)$. The population of mobile agents running in each host denoted by $p_j(t)$ satisfies:

$$km + (1-\xi^{d-1})[1-1/(n-n\xi)](D_j-1)km \le p_j(t) \le km$$
$$+ (1-\xi^{d-1})[1-1/(n-n\xi)](D_j-1)km,$$

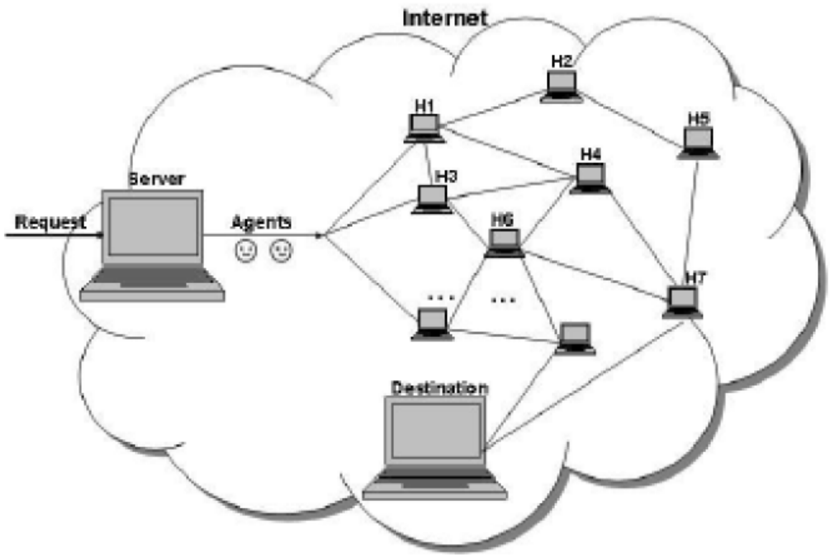
where $\xi = \|A\|_1 = \max_{1\le j\le n}\|a_j\|_1$, $\zeta = \min_{1\le j\le n}\|a_j\|_1$, a_j is the jth column of matrix A, and D_j is the degree of the jth host.

The rest of this paper is organised as follows. Section 2 introduces some background information regarding our analysis. Section 3 presents some analytical results for mobile agents, including the probability of success and the population of agents. Section 5 concludes the paper.

2 Model

As we know, for one online purchase, the buyer keys an order, or goods list, in one server. The order may ask for information such as price, quality, transport cost, etc. of the goods; it might ask for goods with an upper price limit, for goods directly from pre-pointed shops, or some combination of requests. When a server receives an order for an information search, it will generate a number of mobile agents and dispatch them into the network. Those agents will roam the network until they find their appropriate destination shops, searching for relevant information, and sending it back to the server. Figure 1 shows an example of this process.

Figure 1 An example of the searching process

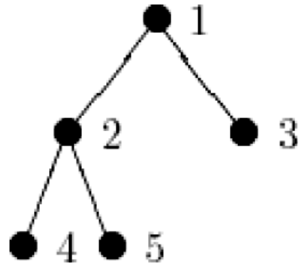


The essential idea of our model can be summarised as follows: once a request is received from a server, the server will generate a number of mobile agents. These agents will then move out from the server searching for appropriate destination shops. From each host, the agent will randomly select a neighbouring host to jump to. Considering that there may be some malicious hosts in the network, we assume that it is possible for an agent to be eaten by its current host. Once an agent finds the address of the destination host, a message is sent back to the server following the same path, leaving marks on the hosts along its path. Our analysis makes the following assumptions:

- There are n nodes in the network.
- At any time t, the expected number of requests keyed in one host is m. Once a request arrives, k agents are created and sent out into the network.
- When an agent reaches a host, it will check whether that current host is its destination. If the agent cannot find its destination in the current host, it will jump to any of the neighbouring hosts or die in the current host, with the same probability.
- Once an agent reaches its destination, it submits its goods list to the host and dies. After the host fulfils the relevant requirements, a new agent is generated and dispatched to the server with the resulting information.
- To prevent the user from waiting too long, and to reduce unnecessary searching in the network, we further assume that if an agent cannot find its destination in d jumps, it will die.

In the following segment, we introduce some of the notations and definitions used in this paper. Since the topology of a network can be decided uniquely by its connectivity matrix, the connectivity matrix plays an important role in network management. In this paper, we apply the connectivity matrix for our analysis. For this paper, we assume that there are n nodes in the network, and the network topology is a connected graph so that there is at least one path between any two hosts. Matrix $C = (c_{ij})_{n \times n}$ is the connectivity matrix that describes the connectivity of the graph, i.e., if there is a direct link between host i and host j, $c_{ij} = c_{ji} = 1$. Otherwise, $c_{ij} = 0$. Let c_j be the jth column vector of matrix C: $C = (c_1, c_2, \ldots, c_n)$. $D_j = \|c_j\|_1 = \sum_{i=1}^{n} |c_{ij}|$, $\sigma_1 = \max_{1 \le j \le n} D_j$, $\sigma_n = \min_{1 \le j \le n} D_j$. $D = \mathrm{diag}(D_1, D_2, \ldots, D_n)$ is a diagonal matrix. It is easy to see that D_j is the number of neighbouring hosts of the jth host including itself, and $\|C\|_1 = \max_{1 \le j \le n} \|c_j\|_1 = \sigma_1$. For example, suppose that the graphical structure of a network is as shown in Figure 2:

Figure 2 An example of a small network



Accordingly, $n = 5$, $\sigma_1 = 4$, $\sigma_n = 2$, matrix C and matrix D can be given as follows:

$$C = \begin{pmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{pmatrix} \quad D = \begin{pmatrix} 3 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 2 \end{pmatrix}$$

3 Analysis

In this section, we present an analysis of the probability of success and the population distribution of mobile agents.

3.1 The probability of success

First, we assume that the lengths of each link between two nodes are the same, and the probability that an agent can find its destination at the current host is $1/n$, then the searching time is determined by the number of jumps. Assuming that the probability of jumping to any neighbouring host or die is the same, we have the following theorem for the probability that an agent can find the destination in d jumps as follows:

Theorem 1: *The probability, $P^*(d)$, that at least one agent among the k agents can find the destination in d jumps satisfies the following inequalities such as:*

$$P^*(d) \le 1 - \left[1 - \frac{1}{n} \cdot \frac{1-x^d}{1-x}\right]^k \quad \text{and} \quad \lim_{d\to\infty} P^*(d) \le 1 - \left(\frac{n-1}{n+\sigma_1 - 1}\right)^k$$

where $x = (1-1/n)(1-1/\sigma_1)$.

Proof: Denote the sequence number of the node that the agent entered at the dth jump by J_d and the probability that an agent can find its destination at the dth jump by $P(d)$. The probability that an agent can find its destination at the first jump is $P(1) = 1/n$ and the probability it cannot find the destination is $1 - 1/n$ If the agent cannot find its destination, the probability that it can jump out and continue to search is $(1-1/n)[(D_{J_1} - 1)/D_{J_1}]$, and the probability that it can find its destination at the second jump is $P(2) = (1/n)(1-1/n)[(D_{J_1} - 1)/D_{J_1}]$; the probability that it cannot find its object at the second jump is $(1-1/n)^2[(D_{J_1} - 1)/D_{J_1}]$. If the agent cannot find its destination at the second jump, the probability that it takes the third jump is $(1-1/n)^2[(D_{J_1} - 1)/D_{J_1}][(D_{J_2} - 1)/D_{J_2}]$, and therefore we have $P(3) = (1/n)(1-1/n)^2[(D_{J_1} - 1)/D_{J_1}][(D_{J_2} - 1)/D_{J_2}]$. Similarly, the probability that an agent can find its destination host at the dth jump is

$$P(d)=\frac{1}{n}\left(1-\frac{1}{n}\right)^{d-1}\prod_{i=1}^{d-1}\frac{D_{J_i}-1}{D_{J_i}}\le\frac{1}{n}\left(1-\frac{1}{n}\right)^{d-1}\left(\frac{\sigma_1-1}{\sigma_1}\right)^{d-1}$$

So the probability, $P^*(d)$, that at least one agent among k agents can find the destination host in d jumps satisfies the following:

$$P^*(d)=\sum_{s=1}^{k}C_k^s\left[\sum_{t=1}^{d}P(t)\right]^s\left[1-\sum_{t=1}^{d}P(t)\right]^{k-s}=1-\left[1-\sum_{t=1}^{d}P(t)\right]^k$$

Due to $(1-1/n)(1-1/\sigma_1)<1,$ we have

$$\sum_{t=1}^{d}P(t)\le\sum_{t=1}^{d}\frac{1}{n}\left(1-\frac{1}{n}\right)^{t-1}\left(\frac{\sigma_1-1}{\sigma_1}\right)^{t-1}=\frac{1}{n}\cdot\frac{\left[\left(1-\frac{1}{n}\right)\left(1-\frac{1}{\sigma_1}\right)\right]^d}{1-\left(1-\frac{1}{n}\right)\left(1-\frac{1}{\sigma_1}\right)}$$

Let $x=(1-1/n)(1-1/\sigma_1),$ then

$$\sum_{t=1}^{d}P(t)\le\frac{1}{n}\cdot\frac{1-x^d}{1-x}$$

Therefore, we have

$$P^*(d)\le1-\left[1-\frac{1}{n}\cdot\frac{1-x^d}{1-x}\right]^k$$

and furthermore

$$\lim_{d\to\infty}P^*(d)\le1-\left(\frac{n-1}{n+\sigma_1-1}\right)^k$$

Hence, the theorem is proven.

The probability that an agent can find its destination is decided by the topology of the network and parameters k and d, theoretical results that coincide with practice. From this theorem, we can easily determine that the probability that none of those k agents can find the destination is equal to $[(n-1)/(n+\sigma_1-1)]^k$, and the probability that all the k agents can find the destination is equal to $[\sigma_1/(n+\sigma_1-1)]^k$.

3.2 *The population of agents*

Assume that at time $t-1$, there are $p_i(t-1)$ agents in the ith host. These agents search for the destination locally; the expected number of agents that cannot find the destination is equal to $(1-1/n)p_i(t-1)$, and the expected number of mobile agents moving from the ith host to the jth host is equal to $(1-1/n)p_i(t-1)/D_i$. The total number of agents moving to the jth host at time t is $\sum_{i\in NB(j)}(1-1/n)(1/D_i)p_i(t-1)$, where $NB(j)$ denotes the host set make up of the neighbouring hosts of the jth host. Considering the new generated agents in the jth host at time t, we have the following equation:

$$p_j(t) = km + \sum_{i \in NB(j)} \left(1 - \frac{1}{n}\right)\left(\frac{1}{D_i}\right) p_i(t-1)$$

where m is the average number of requests initiated at time t at a host, and k is the number of agents generated per request. Let $\vec{p}(t) = (p_1(t), p_2(t), \ldots, p_n(t))^T$, and $\vec{e}$ be a vector of which all the elements are one, we can express the population distribution of mobile agents in vector-matrix format:

$$\vec{p}(t) = A\vec{p}(t-1) + km\vec{e}$$

where $A = (1 - 1/n)(C - I)D^{-1} = (a_1, a_2, \ldots, a_n)$ is a matrix, a_j is the jth column vector of A. Obviously, we have $\|a_j\|_1 = (1 - 1/n)[(D_j - 1)/D_j]$

Theorem 2: *The distribution of mobile agents can be expressed as follows:*

$$\vec{p}(t) = \begin{cases} 0 & t = 0 \\ (I + A + A^2 + \cdots + A^{t-1})km\vec{e} & 0 < t \le d \\ (I + A + A^2 + \cdots + A^{d-1})km\vec{e} & t > d \end{cases}$$

Proof: First, if the distribution of mobile agents created at time 0 is $\vec{p}(0)$, then after time t, the distribution of the surviving agents is $A^T \vec{p}(0)$. Second, by the recursive formula of the distribution of mobile agents, $\vec{p}(\mathrm{t}) = km\vec{e} + A\vec{p}(t-1)$, and the assumption that if an agent cannot find the destination in d steps, it will die, we have

- If $t \le d$,

$$\begin{aligned} \vec{p}(\mathrm{t}) &= km\vec{e} + A\vec{p}(t-1), \\ &= km\vec{e} + A(km\vec{e} + A\vec{p}(t-2) \\ &= (I + A)(km\vec{e} + A^2\vec{p}(t-2) \\ &= \ldots \\ &= (I + A + A^{t-1})km\vec{e} + A^t\vec{p}(0) \end{aligned}$$

  As we had assumed that the initial population of mobile agents $\vec{p}(0) = 0$, the result for $t < d$ is proven.

- If $t > d$, then at time t, all the survival agents generated at time $t \le d$ die; so the distribution of agents under this case can be illustrated as:

$$\begin{aligned}
\vec{p}(t) &= km\vec{e} + A\vec{p}(t-1) - A^d km\vec{e} \\
&= (I + A + \cdots + A^{t-d-1})km\vec{e} + A^{t-d}\vec{p}(d) - (I + A + \cdots + A^{t-d-1})A^d km\vec{e} \\
&= (I + A + \cdots + A^{t-d-1})km\vec{e} + A^{t-d}[(I + A + \cdots + A^{d-1})km\vec{e} + A^d\vec{p}(0)] \\
&\quad -(I + A + \cdots + A^{t-d-1})A^d km\vec{e} \\
&= (I + A + \cdots + A^{d-1})km\vec{e}
\end{aligned}$$

Hence, the theorem is proven.

From this theorem, we can easily see that the distribution of mobile agents will not exceed $(I + A + \cdots + A^{d-1})km\vec{e}$, i.e., the number of mobile agents in our model has an upper limitation.

Since mobile agents are frequently generated and dispatched to the network, it is important to be able to estimate the maximum number of mobile agents running in the network and in each host. When there are too many agents in the network, they will introduce too much computational overhead in the host machines, which will eventually become very busy and indirectly block the network traffic.

Regarding the number of agents running in the network, we have the following theorem.

Theorem 3: *The maximum number of agents running in the network with finite d jumps can be estimated as follows:*

$$\sum_{j=1}^{n} P_j(t) \le \frac{nkm(1-x^d)}{1-x} \text{ and } \lim_{d\to\infty} \sum_{j=1}^{n} P_j(t) \le \frac{n^2\sigma_1 km}{n+\sigma_1-1}$$

where $x = \|A\|_1 = (1-1/n)\cdot[(\sigma_I - 1)/\sigma_I]$.

Proof: By the definition of matrix 1-norm, we have

$$\sum_{j=1}^{n} p_j(t) = \|\vec{p}(t)\|_1 \le \|I + A + \cdots + A^{d-1}\|_1 \cdot \|km\vec{e}\|_1 \le nkm \cdot \sum_{i=0}^{d-1} (\|A\|_1)^i \le \frac{nkm}{1-\|A\|_1}.$$

Due to $\|A\|_1 = (1-1/n)\cdot[(\sigma_I - 1)/\sigma_I] < 1$, denoted $\|A\|_1$ by x for short, it is easy to prove that

$$\sum_{j=1}^{n} p_j(t) \le \frac{nkm(1-x^d)}{1-x} \text{ and } \lim_{d\to\infty} \sum_{j=1}^{n} p_j(t) \le \frac{n^2\sigma_1 km}{n+\sigma_1-1}$$

Hence, the theorem is proven.

Define $\xi = \max_{1\le j\le n} \|a_j\|_1 = (1-1/n)[(\sigma_I - 1)/\sigma_I]$, and $\zeta = \min_{1\le j\le n} \|a_j\|_1 = (1-1/n)[(\sigma_n - 1)/\sigma_n]$.

For the number of agents running in a host, we have the following theorem:

Theorem 4: *The number of agents running in the jth host can be estimated as follows:*

$$\begin{cases} km+(1-\xi^{t-1})\left(1-\dfrac{1}{n-n\xi}\right)(D_j-1)km \le p_j(t) \le km+(1-\zeta^{t-1})\times \\ \left(1-\dfrac{1}{n-n\zeta}\right)(D_j-1)km \quad 0<t\le d \\ km+(1-\xi^{d-1})\left(1-\dfrac{1}{n-n\xi}\right)(D_j-1)km \le p_j(t) \le km+(1-\zeta^{d-1})\times \\ \left(1-\dfrac{1}{n-n\zeta}\right)(D_j-1)km \quad t>d \end{cases}$$

Proof: Assume that *NB*(*j*) is a set constituted by all the neighbouring hosts of the *j*th host. By recursion, we have

$$p_j(0)=0$$

$$p_j(1)=km$$

$$p_j(2)=km+\sum_{i\in NB(j)}(1-1/n)p_i(1)/D_i$$

$$\le km+\sum_{i\in NB(j)}\left(km-\frac{km}{n}-\zeta km\right)$$

$$=D_jkm+\sum_{i\in NB(j)}\left(km-\frac{(D_j-1)km}{n}-\zeta(D_j-1)km\right)$$

$$p_j(3)=km+\sum_{i\in NB(j)}\left[\left(1-\frac{1}{n}\right)p_i(2)\right]\Big/D_i$$

$$\le km+\sum_{i\in NB(j)}\left[\left(1-\frac{1}{n}\right)km-\frac{1}{n}\zeta km-\zeta^2 km\right]$$

$$=D_jkm-\frac{1}{n}(D_j-1)km-\frac{1}{n}\zeta(D_j-1)km-\zeta^2(D_j-1)km$$

$$p_j(t)\le D_jkm-\frac{1}{n}(D_j-1)km\sum_{s=0}^{t-2}\zeta^s-\zeta^{t-1}(D_j-1)km$$

$$=km+(1-\xi^{t-1})\left[1-\frac{1}{n(1-\xi)}\right](D_j-1)km$$

Similarly, we have

$$p_j(t)\ge D_jkm-\frac{1}{n}(D_j-1)km\sum_{s=0}^{t-2}\xi^s-\xi^{t-1}(D_j-1)km$$

$$= km + (1-\xi^{t-1})\left[1 - \frac{1}{n(1-\xi)}\right](D_j - 1)km$$

When $t \geq d$, since $p_j(t) = (I + A + \cdots + A^{d-1})km\vec{e}$, the bounds are fixed and irrelevant to t. Hence, the theorem is proven.

It is easy to see that $p_j(t)$ is related to the values of k, m, t and the topology of the network, which agrees with our intuition. When t is big enough, both ζ and ξ approximate to zero, then we have the following corollary:

Corollary 1: *If t is big enough, then we have:*

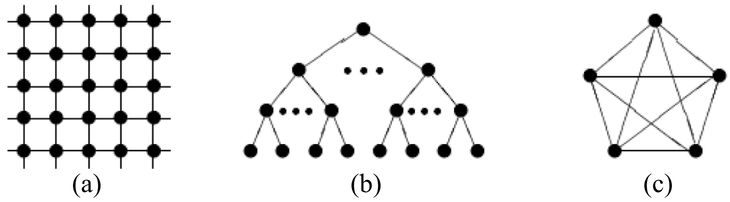
$$p_j(t) \approx km + \left[1 - \frac{1}{n}\right](D_j - 1)km$$

Furthermore, since both $1-\zeta^{d-1}$ and $1-1/(n(1-\zeta))$ are less than 1, we have $p_j(t) \leq D_j km$, which is a result given in Menczer et al. (2002).

4 Experimental results

In this section, we present some examples to explain the analytical results given in Section 3. We have conducted many experiments for several network topologies and found that our analytical results are reasonable for the different topologies. Here, because of space limitations, we list only the experimental results for three graphs. Figure 3(a) is a square lattice, Figure 3(b) is a Cayley tree with u branches, and Figure 3(c) is a complete graph.

Figure 3 Three kinds of graphs for experiments



4.1 Searching time of per request

In this subsection, all the experimental results conclude the three cases as shown in Figures 4–6. Table 1 presents the experimental results on the upper bound of the survival probability (UBSP) and the lower bound of the successful probability (LBSP); survival probability is defined as the probability that an agent can survive after d jumps and the lower bound of the successful probability is defined as the probability that the destination can be found within d jumps. The results shown in Table 1 are for the case in which $k = 20$. We can see that, when d increases, USBP will decrease and LBSP will increase. As n increases, USBP will increase and LBSP will decrease. Furthermore, if UBSP is very large, the LBSP will increase rapidly with the growth of d, and if USBP is very small, the LBSP will increase slowly; when the LBSP increases too slowly, the process of searching should be shut down. All of these results are coherent with Theorem 1 and its conclusions. Table 2 shows the experimental results on the lower bound on *k* when *d* approximates to infinity. The values in the first row of the table are values of P_0. We can see that the larger the degree of node (the number of neighbouring hosts), the fewer the number of agents needed.

Figure 4 Results for square lattice (Figure 3(a))

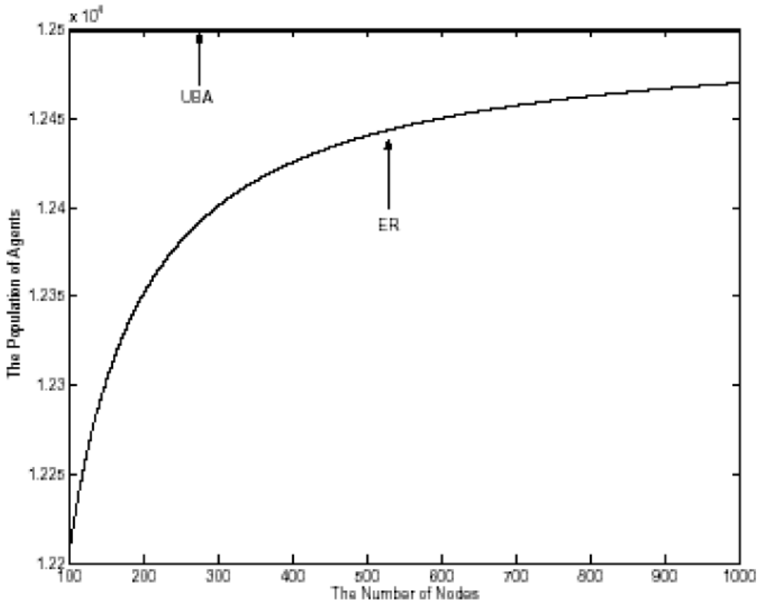


Figure 5 Results for Cayley tree (Figure 3(b))

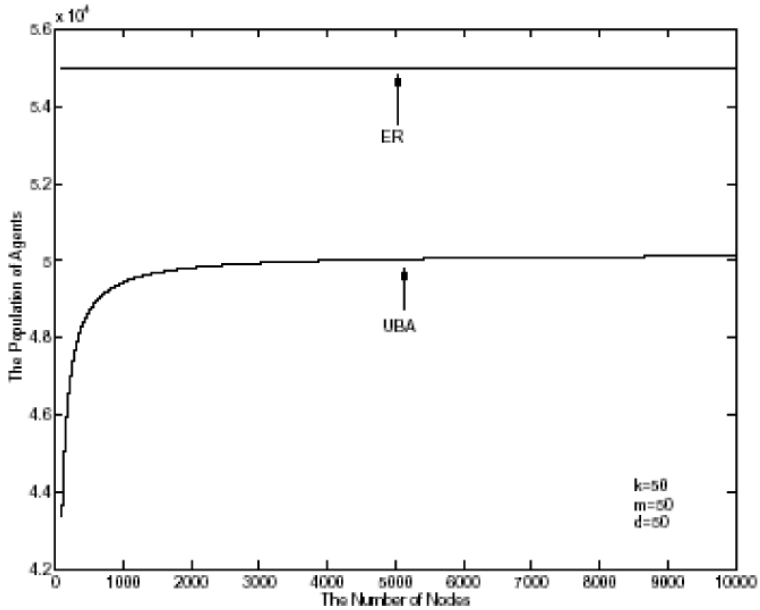


Figure 6 Results for complete graph (Figure 3(c))

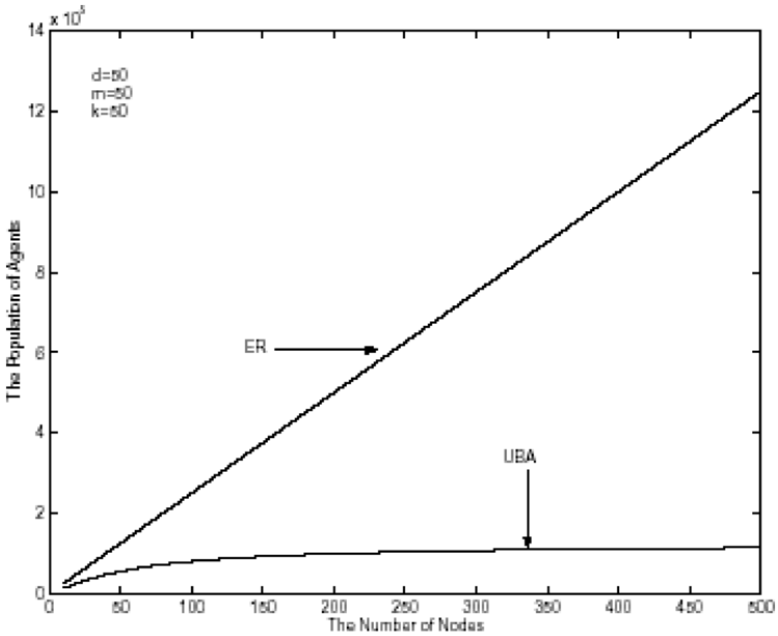


Table 1 The experiment results on UBSP and LBSP

		10		20		50		100	
n		*UBSP*	*LBSP*	*UBSP*	*LBSP*	*UBSP*	*LBSP*	*UBSP*	*LBSP*
200	A	0.1283	0.2549	0.0131	0.2586	1.3955e–05	0.2586	1.5502e–10	0.2586
	B	0.6289	0.5554	0.3756	0.7273	0.0801	0.8449	0.0061	0.8642
	C	0.9137	0.6247	0.8266	0.8519	0.6119	0.9878	0.3707	0.9995
500	A	0.1318	0.112	0.0139	0.1129	1.6174e–05	0.1130	2.0885e–10	0.1130
	B	0.6462	0.2770	0.3978	0.4067	0.0928	0.5315	0.0082	0.5592
	C	0.9646	0.3275	0.9267	0.5441	0.8219	0.8510	0.6727	0.9729
1e3	A	0.1330	0.0572	0.0140	0.0582	1.6987e–05	0.0582	2.3062e–10	0.0582
	B	0.6520	0.1497	0.4054	0.2300	0.0974	0.3170	0.0091	0.3385
	C	0.9822	0.1806	0.9627	0.3273	0.9066	0.6232	0.8203	0.8507
2e3	A	0.1336	0.0290	0.0143	0.0295	1.7409e–05	0.0295	2.4233e–10	0.0295
	B	0.6550	0.0779	0.4093	0.1226	0.0999	0.1740	0.0095	0.1874
	C	0.9910	0.0950	0.9812	0.1805	0.9522	0.3898	0.9057	0.6230
1e4	A	0.1341	0.0059	0.0144	0.0060	1.7753e–05	0.0060	2.5212e–10	0.0060
	B	0.6573	0.0161	0.4124	0.0258	0.1018	0.0376	0.0099	0.0408
	C	0.9982	0.0198	0.9962	0.0392	0.9902	0.0948	0.9804	0.1805
1e6	A	0.1342	5.8958e–05	0.0144	5.9980e–05	1.7840e–05	5.9998e–05	2.5460e–10	5.9998e–05
	B	0.6579	1.6214e–04	0.4132	2.6157e–04	0.1023	3.8330e–04	0.0100	4.1672e–04
	C	0.99982	1.9998e–04	0.99996	3.9992e–04	0.9999	9.9948e–04	0.9998	0.0020

Table 2 The lower bond on k when d approximates to infinity

n	*Case*	*0.1*	*0.2*	*0.3*	*0.4*	*0.5*	*0.6*	*0.7*	*0.8*	*0.9*
200	A	5	9	15	21	28	37	49	65	93
	B	2	3	4	5	7	9	12	16	22
	C	1	1	1	1	1	2	2	3	4
500	A	11	23	36	52	70	92	121	162	231
	B	3	6	9	12	17	22	28	38	54
	C	1	1	1	1	1	2	2	3	4
1000	A	22	45	72	103	139	184	242	323	462
	B	5	11	17	24	32	43	56	74	106
	C	1	1	1	1	1	2	2	3	4
2000	A	43	90	143	205	278	367	482	645	922
	B	10	21	33	47	64	84	110	148	211
	C	1	1	1	1	1	2	2	3	4
10000	A	211	447	714	1022	1387	1833	2409	3220	4606
	B	48	102	163	233	316	417	548	733	1048
	C	1	1	1	1	1	2	2	3	4
1000000	A	2108	44629	71335	102170	138630	183260	240790	321890	460520
	B	479	10143	16213	23220	31507	41650	54727	73157	104660
	C	1	1	1	1	1	2	2	3	4

4.2 The population of agents

In this subsection, we present some experimental results on the population of mobile agents. Figures 4–6 show the results on the population of agents running in a host. Here, we compare our results with that in Menczer et al. (2002). In the figures, UBA refers to our results for the upper bound on the population of agents running in a host, while ER refers to the simulation results of existing work. It is obvious that our results greatly outperform the existing results when the number of neighbouring hosts is not very small.

5 Concluding remarks

In this paper, we addressed the problem of mobile agent-based e-shopping. We analysed the growth of the population of mobile agents and the probability of their success. We obtained the following analytical results:

- The probability of success, $P(d)$, that an agent can find the destination in d jumps is less than $(1/n)[1-(1/n)]^d[1-(1/\sigma_1)]^{d-1}$, where n is the number of hosts in the network, σ_1 is the maximum degree of hosts in the network, and d is the number of jumping hops.

- The probability of success that k agents can find the destination in d jumps is estimated as $P^*(d) \leq 1-\left(n-1/(n+\sigma_1-1)\right)^k$, where k is the number of agents generated per request.
- The population distribution can be expressed as $\vec{p}(t) = (I + A + \cdots + A^{d-1})km\vec{e}$ or $\vec{p}(t) = km\vec{e} + A\vec{p}(t-1)$ when $0 < t \leq d$ and $\vec{p}(t) = (I + A + \cdots + A^{d-1})km\vec{e}$ when $t > d$, where $\vec{p}(t)$ is a population vector, m is the average number of requests entering a host at any time, $\vec{e} = (1,1,\ldots,1)^T$ is a vector, A is a matrix derived from the connectivity matrix, and d is a time bound which is given before the agents start their tasks.
- The total number of agents running in the network is less than $(n^2\sigma_1 km)/(n+\sigma_1-1)$.
- The population of mobile agents running in each host, $p_j(t)$, satisfies

$$km + (1-\xi^{d-1})[1-1(n-n\xi)](D_j-1)km \leq p_j(t) \leq km + (1-\zeta^{d-1})[1-1(n-n\zeta)](D_j-1)km,$$

where $\xi = \|A\|_1 = \max_{1\leq j\leq n}\|a_j\|_1$, $\zeta = \min_{1\leq j\leq n}\|a_j\|_1$, a_j is the jth column of matrix A, and D_j is the degree of the jth host.

6 Implications for management

With dramatic advances in the internet and in the computer industry, computers are no longer isolated number factories. Many new applications, from e-business to e-government and e-education, have been created, thanks to the exponential growth of the internet user base and the widespread popularity of the World Wide Web. Integrating these applications into a unitary e-society will require a common infrastructure for these applications. The advantage of mobile agents is that they provide a single infrastructure within which a wide range of distributed applications can be implemented easily, efficiently, and robustly Gray et al. (2000).

Since mobile agents are the medium for executing various applications, their behaviours are vital for both network performance and user satisfaction. In our study, we found that though a single mobile agent may act randomly, the behaviour of all mobile agents as a whole has some stochastic properties. In this paper, we analysed the probability of success and the population distribution as a way to estimate user satisfaction and network performance. Since there is a trade-off between these two factors, managers can achieve maximum benefits by tuning the relevant parameters in our analysis according to the users' requirements and the characteristics of the network.

Acknowledgement

The authors appreciate the help of Judith Steeh, the Head Editor of Technical Communication Program, Japan Advanced Institute of Science and Technology, in editing the paper. This work was supported by the Japan Society for the Promotion of Science (JSPS) Grant-in-Aid for Scientific Research under its grant for Research in Special Research Domains and a grant from the Funai Foundation.

References

Ahola, H., Oinas-Kukkonen, H. and Koivumäki, T. (2000) 'Customer delivered value in a web-based supermarket', *Proc. of the 33rd Haxaii International Conference on System Sciences-2000*.

Barain, M. (1998–2003) *How E-commerce Works*, HowStuffWorks, Inc.

Brazier, F.M.T., Overeinder, B.J., Steen, M. and Wijngaards, N.J.E. (2002) 'Agent factory: generative migration of mobile agents in heterogeneous environment', *Proc. of the ACM Symposium on Applied Computing*, *SAC02*, pp.101–106.

Brewington, B., Gray, R., Moizumi, K., Kotz, D., Cybenko, G. and Rus, D. (1999) 'Mobile agents in distributed information retrieval', in Klusch, M. (Ed.): *Intelligent Information Agents: Agents-Based Information Discovery and Management on the Internet*, Springer-Verlag, Berlin, Chapter 15, pp.355–395.

Claessens, J., Preneel, B. and Vandewalle, J. (2003) '(How) can mobile agents do secure electronic transactions on untrusted hosts? A survey of the security issues and the current solution', *ACM Transactions on Internet Technology*, Vol. 3, No. 1, pp.28–48.

Feldman, S. (2000) 'Electronic marketplaces', *IEEE Internet Computing*, pp.93–95, http://www.brint.com/papers/kmebiz/kmebiz.html.

Gallaugher, J.M. (2002) 'E-commerce and the undulating distribution channel', *Communications of the ACM*, Vol. 45, No. 7, pp.89–93.

Garey, M. and Johnson, D. (1979) *Computers and Intractability: A Guide to the Theory of Np-Completeness*, Freeman.

Gray, R., Kotz, D., Cybenko, G. and Rus, C. (2000) *Mobile Agents: Motivations and State-of-art Systems*, Technical Report TR 2000-365, Department of Computer Science, Dartmouth College, April.

He, M., Jennings, N.R. and Leung, H. (2003) 'On agent-mediated electronic commerce', *IEEE Transactions on Knowledge Engineering e*, Vol. 15, No. 4, pp.985–1003.

Kim, P.J. and Noh, Y.J. (2003) 'Mobile agent system architecture for supporting mobile market application service in mobile computing environment', *Proc. of the 2003 Int'l Conf. on Geometric Modeling and Graphics*, *GMAG'03*.

Kim, S.H. and Robertazzi, T.G. (2000) *Mobile Agent Modeling*, SUNY at Stony Brook Technical Report 786.

Kowalczyk, R., Braun, P., Eismann, J., Franczyk, B., Rossak, W.R. and Speck, A. (2002) 'Intermarket: towards intelligent mobile agent e-marketplaces', *Proc. of the 9th Annual IEEE In'l Conf. and Wksp on the Engineering of Computer-Based Systems, ECBS 2002*.

Lange, D. and Oshima, M. (1999) 'Seven good reasons for mobile agents', *Communications of the ACM*, Vol. 42, pp.88, 89.

Li, T. and Lam, K. (2002) 'An optimal location update and searching algorithm for tracking mobile agent', *Proc. of AAMAS'02*, pp.15–19.

Marques, P.J., Silva, L.M. and Silva, J.G. (1999) 'Security mechanisms for using mobile agents in electronic commerce', *Proc. of the 18th IEEE Symp. on Reliable Distributed Systems*, October, pp.18–21.

Menczer, F., Monge, A.E. and Street, W.N. (2002) 'Adaptive assistants for customized e-shopping', *IEEE Intelligent Systems*, November–December, pp.12–19.

Milojicic, D. (2000) 'Guest editor's introduction: agent systems and applications', *IEEE Concurrency*, Vol. 8, No. 2, pp.22, 23, 69.

Qu, W., Shen, H. and Sum, J. (2003) 'New analysis on mobile agent based network routing', *Proc. of the 3rd Int'l Conf. on Hybrid Intelligent Systems (HIS'0)*, pp.769–778.

Sum, J., Shen, H., Leung, C.S. and Young, G. (2003) 'Analysis on mobile-agent based algorithm for network routing and management', *IEEE Transaction on Parallel and Distributed Systems*, Vol. 14, No. 3, pp.193–202.

Tang, L. and Pagurek, B. (2002) 'A comparative evaluation of mobile agent performance for network management', *Proc. of the 9th Annual IEEE Int'l Conf. and Wksp. on the Engineering of Computer-Based Systems*, ECBS'02.

Wagner, C. and Turban, E. (2002) 'Are intelligent e-commerce agents partners or predators?', *Communications of the ACM*, Vol. 45, No. 5, pp.84–90.

Wang, Y. (1995) 'Dispatching multiple mobile agents in parallel for visiting e-shops', *Proc. of the 3rd Int'l Conf. on Mobile Data Management*, MDM'02.