Regularization Parameter Selection for Faulty Neural Networks

Hong-jiang Wang, Fei Ji, Gang Wei, Chi-Sing Leung, and Pui-Fai Sum

***Abstract*—**Regularization techniques have attracted many researches in the past decades. Most focus on designing the regularization term, and few on the optimal regularization parameter selection, especially for faulty neural networks. As is known that in the real world, the node faults often inevitably take place, which would lead to many faulty network patterns. If employing the conventional method, i.e., the test set method or cross-validation, to find the optimal regularization parameter, it will cost a lot of time. Moreover, although some statistic methods have been proposed, almost of them aim at the fault-free networks. Thus, in the paper, a MPE formula is derived to evaluate the mean prediction error in the multi-node open fault situations and then used to select the optimal regularization parameter. Experiment results have shown that the optimal parameter value selected by our proposed formula is very close to the actual one, chosen by the conventional test set method. Our contribution is that our proposed MPE formula can be used in choosing the regularization parameter instead of the test set method for faulty neural networks with multi-node open fault.

***Keywords*—**Faulty neural networks, multi-node open fault, regularization parameter, mean prediction error.

I. Introduction

As is known that the node faults inevitably take place in the real application for neural networks, especially in VLSI [1]. Without the special care, the fault situation could result in drastic performance degradation [2]. Due to the simplicity of computation and structure, regularization methods have been considered as one of the most effective techniques for improving the fault tolerant through adding one regularization term, consisting of regularization parameter and regularization matrix, in the objective function [3]. The regularization parameter plays an important role in the regularization method and controls the overfitting or underfitting of the network learning process. If the improper regularization parameter is selected, the network learning performance, i.e., generalization, will suffer from the drastic degradation [4]. Thus, it is necessary to choose a proper regularization parameter for improving the generalization performance of faulty neural network with the regularization method.

Traditional methods of choosing the regularization parameter is the test set method or cross-validation [5]-[7], i.e., calculating the prediction error with the testing or training dataset for each the candidate regularization parameter and then selecting the optimal regularization parameter with the minimum prediction error value. However, when the network suffers from the multi-node open fault, i.e., multiple nodes synchronously take no effect in the training process [8], the above operation process has to be repeated for each faulty network pattern and then the mean prediction error (MPE) is calculated, which will cost a lot of dealing time. In addition, though some statistical methods have been proposed to evaluate the mean prediction error, e.g., Moody's GPE [9], most of them aim at the fault-free neural networks. To our best knowledge, no literatures about regularization parameter selection for faulty neural networks have been published.

Thus, the paper derive a formula, called as MPE formula, to evaluate the mean prediction error for faulty neural networks in multi-node open fault conditions based on Moody's thinking and then use it to choose the optimal regularization parameter. Simulation results show that our proposed MPE formula can quickly find the optimal regularization parameter, which is very close to the actual one by the test set method.

II. Faulty Neural networks

In the section, we shall present the basic framework of the faulty neural networks.

A. RBF model

We consider the RBF network model as an example. For clarity, the bold letter denotes the vector in the paper. Thus, the unknown function mapping $f(\cdot)$ can be approximated as [10]

$$f(\mathbf{x}) \approx \hat{f}(\mathbf{x}, \mathbf{w}) = \sum_{j=1}^{M} w_j \phi_j(\mathbf{x}) = \mathbf{\Phi}^T(\mathbf{x})\mathbf{w} \text{ ,} \tag{1}$$

where $\mathbf{x}=\left[x_1,x_2,\cdots,x_N\right]$ is the training input dataset, those corresponding output dataset $\mathbf{y}=[y_1, y_2, \cdots, y_N]$ can be written by $\mathbf{y}=f(\mathbf{x})+\mathbf{n}$, $\mathbf{n}$ is the independent identity distribution Gaussian random noise vector with mean zero and variance σ_n^2 . $\mathbf{w}=[w_1, w_2, \cdots, w_M]$ is the RBF weight vector, and $\phi_j(\mathbf{x}) = \exp\left(\left\|\mathbf{x}-c_j\right\|^2 / \Delta\right)$ is the j th kernel function of RBF, $\mathbf{\Phi}(\mathbf{x})=[\phi_1(\mathbf{x}), \phi_2(\mathbf{x}), \cdots, \phi_M(\mathbf{x})]$, $\|\cdot\|$ is the matrix norm,

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c_j 's are the center of RBF, Δ is the kernel width of RBF.

B. Faulty model

The existing literatures often discuss two kinds of fault models, i.e., multi-node open fault [8] and multiplicative weight noise [11], [12]. In the paper, we mainly consider the multi-node open fault situation, where the node fault means that the corresponding weights are equal to zero. Therefore, the faulty weight vector is

$$\widehat{\mathbf{w}} = \mathbf{b} \otimes \mathbf{w}\,, \tag{2}$$

where $\otimes$ is the element-wise multiplication operator, $\mathbf{b} = [b_1, b_2, \cdots, b_M]^T$ is the node fault vector with $b_i \in [0,1]$. Moreover, when $b_i = 0$, the i th node is out of work. Otherwise, the i th one works. p is the node fault rate with the following relation

$$\langle b_i b_{i'} \rangle = \begin{cases} 1-p, \text{ if } i = i' \\ (1-p)^2, \text{ if } i \neq i' \end{cases}. \tag{3}$$

To enhance the fault tolerant of the faulty neural networks, some methods have been proposed, such as injecting weight noise during training [13], adding the node redundancy [14] and regularization [15] and so on.

C. Regularization learning

Generally speaking, the regularization method employs a regularization term to limit weight magnitude since the large weights have a big effect on the output of the neural network. As one of the most common regularizers, the weight decay has been widely researched and applied in the past years. Studies have shown that weight decay regularizer to some extend can improve the fault tolerant in the faulty neural networks, but it is not the best one [8]. Hence, Leung et al. have developed a new regularizer, which has a better ability to tolerate the multi-node open fault than others, e.g., the standard weight decay [16].

III. Mean Prediction Error

The regularizer for faulty RBF neural networks with multi-node open fault proposed by Leung et al. is $\lambda \mathbf{w}^T \mathbf{R} \mathbf{w}$, where $\mathbf{R} = \mathbf{G} - \mathbf{H}_\phi$ is the regularization matrix, λ is the regularization parameter, $\mathbf{H}_\phi = \frac{1}{N}\sum_{j=1}^{N} \phi(x_j)\phi^T(x_j)$ and $\mathbf{G} = \mathrm{diag}(\mathbf{H}_\phi)$. Therefore, the error objective function for faulty neural networks with multi-node open fault can be expressed as

$$E(\mathbf{w}, \lambda) = \frac{1}{N}\sum_{i=1}^{N}\left(y_i - \mathbf{\Phi}^T(x_i)\mathbf{w}\right)^2 + \lambda \mathbf{w}^T \mathbf{R}\mathbf{w}\,, \tag{4}$$

and the training error $E_{train}(\mathbf{w})$ is

$$E_{train}(\mathbf{w}) = \frac{1}{N}\sum_{i=1}^{N}\left(y_i - \mathbf{\Phi}^T(x_i)\mathbf{w}\right)^2 . \tag{5}$$

Following the Moody's thinking [9], we can know that the mean testing error for faulty networks $\overline{E}_{test}(\mathbf{w},\mathbf{b})$ is

$$\overline{E}_{test}(\mathbf{w},\mathbf{b}) = \overline{E}_{train}(\mathbf{w},\mathbf{b}) + 2S_e\frac{M_{eff}}{N}\,, \tag{6}$$

where M_{eff} is the effective number of weights in the RBF model, S_e is the measured noise variance, the first term is the mean training error for faulty networks.

Moreover, according to the definition of M_{eff} in [9], we get

$$M_{eff} = \frac{N}{2}\sum_{i=1}^{N}\sum_{\alpha=1}^{M}\sum_{\beta=1}^{M} T_{i\alpha}U_{\alpha\beta}^{-1}T_{\beta i}\,, \tag{7}$$

where

$$T_{i\alpha} = \frac{\partial^2 E_{train}}{\partial y_i \partial w_\alpha} = -\frac{2}{N}\phi_\alpha(x_i)\,,$$

$$T_{\beta i} = \frac{\partial^2 E_{train}}{\partial w_\beta \partial y_i} = -\frac{2}{N}\phi_\beta(x_i)\,,$$

$$U_{\alpha\beta} = \frac{\partial^2 E(\mathbf{w},\mathbf{b})}{\partial w_\alpha \partial w_\beta} = 2\left(\mathbf{H}_\phi + p\mathbf{R}\right)\cdot$$

Therefore,

$$M_{eff} = \mathrm{trace}\left(\mathbf{H}_\phi\left(\mathbf{H}_\phi + p\mathbf{R}\right)^{-1}\right)\cdot \tag{8}$$

In addition, the mean train error for faulty neural networks can be expressed as

$$\begin{aligned} \overline{E}_{train}(\mathbf{w},\mathbf{b}) &= \left\langle \frac{1}{N}\sum_{i=1}^{N}\left(y_i - \mathbf{\Phi}^T(x_i)\widetilde{\mathbf{w}}\right)^2 \right\rangle \\ &= \left\langle \frac{1}{N}\sum_{i=1}^{N}\left(y_i - \mathbf{\Phi}^T(x_i)(\mathbf{b}\otimes\mathbf{w})\right)^2 \right\rangle \\ &= \frac{1}{N}\sum_{i=1}^{N} y_i^2 - 2(1-p)\cdot\frac{1}{N}\sum_{i=1}^{N}\left(y_i\cdot\mathbf{\Phi}^T(x_i)\mathbf{w}\right) \\ &\quad +(1-p)\mathbf{w}^T\left(\mathbf{H}_\phi + p\mathbf{R}\right)\mathbf{w} \end{aligned} \tag{9}$$

and the mean training error for fault-free neural networks is

$$\begin{aligned} E_{train}(\mathbf{w}) &= \left\langle \frac{1}{N}\sum_{i=1}^{N}\left(y_i - \mathbf{\Phi}^T(x_i)\mathbf{w}\right)^2 \right\rangle \\ &= \frac{1}{N}\sum_{i=1}^{N} y_i^2 - 2\cdot\frac{1}{N}\sum_{i=1}^{N}\left(y_i\cdot\mathbf{\Phi}^T(x_i)\mathbf{w}\right) + \mathbf{w}^T\mathbf{H}_\phi\mathbf{w} \end{aligned} \tag{10}$$

Thus, comparing (9) and (10), we can get

$$\begin{aligned} \overline{E}_{train}(\mathbf{w},\mathbf{b}) &= (1-p)E_{train}(\mathbf{w}) + p\cdot\frac{1}{N}\sum_{i=1}^{N} y_i^2 \, . \\ &\quad +\left(p-p^2\right)\mathbf{w}^T\mathbf{R}\mathbf{w} \end{aligned} \tag{11}$$

Thereby, substituting (8) and (11) into (6), we have

$$\begin{aligned} MPE = \overline{E}_{test}(\mathbf{w},\mathbf{b}) &= (1-p)E_{train}(\mathbf{w}) \\ &\quad +p\cdot\frac{1}{N}\sum_{i=1}^{N} y_i^2 + \left(p-p^2\right)\mathbf{w}^T\mathbf{R}\mathbf{w} \\ &\quad +\frac{2S_e}{N}\mathrm{trace}\left(\mathbf{H}_\phi\left(\mathbf{H}_\phi + p\mathbf{R}\right)^{-1}\right) \end{aligned} \tag{12}$$

Using the MPE formula, we can rapidly obtain the MPE value and select the optimal regularization parameter.

IV. SIMULATIONS

To test our theoretic results, two function approximation examples have been employed, i.e., sinc function and nonlinear autoregressive (NAR) time series prediction problems. In the simulations, firstly, we use our proposed formula method, MPE formula, and the test set method to calculate the testing error values of each the candidate regularization parameter λ, and then take the optimal parameter value with the minimal testing error value, respectively. Finally, two results have been compared to verify our theoretic results. Therefore, to get the best test results, it is important to select the approximate candidate regularization parameter. Taking the tradeoff of the complexity and accuracy, we set the candidate regularization parameter as $\lambda = \left[10^{-4}, 10^{-3.95}, \cdots, 10^{0}\right]$. Moreover, we random generate 10000 faulty networks to calculate the actual MPE with the test set method. In addition, the variance of measured noise S_e can be obtained using the Fedorov's method [17], i.e.,

$$S_e = \frac{\left\| \mathrm{y} - \Phi^T(\mathrm{x})\mathrm{w} \right\|^2}{N - M}. \tag{10}$$

A. One-dimension function approximation

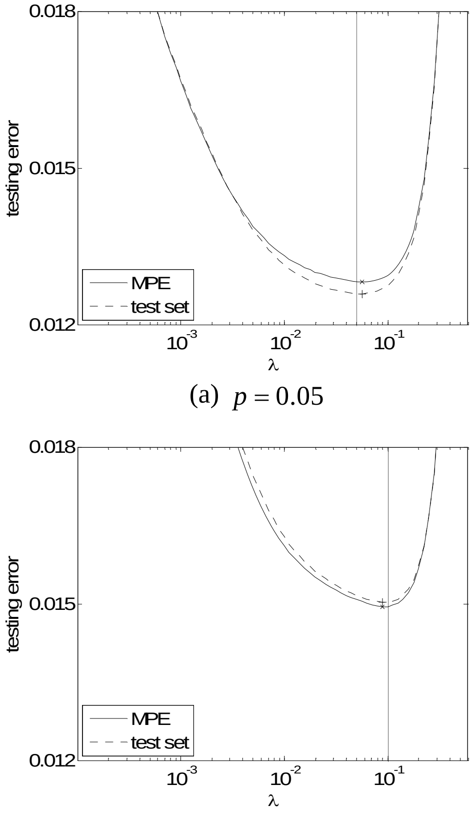
The sinc function approximation problem is a benchmark one-dimension function learning example [18]. The function relation equation can be generated by

$$\mathbf{y} = \mathrm{sinc}(\mathbf{x}) + \mathbf{n}. \tag{11}$$

where $\mathbf{n}$ is the independent identity distribution Gaussian random noise vector with mean zero and variance 0.01. In simulation, 200 training dataset and 1000 testing dataset are generated. The RBF network model has 37 nodes, which are taken uniformly between -4.5 and 4.5. In addition, the kernel width value of RBF network is 0.49 according to Sum's method in [19]. Our task is to find and compare the optimal regularization parameters for two methods, i.e., MPE formula and the test set method.

Fig.1 has shown simulation results for the sinc function approximation example using two evaluation methods, i.e., MPE formula and test set method. From the figures, we can see that the optimal regularization parameters selected by the two methods are very close. For example, when the fault rate is equal to 0.05, the chosen regularization parameter for two methods are both about 0.056234. Moreover, we have repeated the simulations for 50 times and found the range of optimal regularization parameter values chosen by the MPE formula is around from 0.050119 to 0.063096.

In addition, to deeply affirm our results, we set the fault rate as 0.1 and repeat the above training and testing process. The similar curve shapes can be observed in fig.1(b), i.e., the lowest points of two curves almost are in one vertical line as well. Therefore, we can find that no matter how the fault rate varies, the optimal regularization parameter selected by our proposed formula is very close to the one by the conventional method, i.e., the test set method.



(a) $p = 0.05$

(b) $p = 0.1$

Fig. 1 Optimal regularization parameter selection for sinc function approximation example with the different methods, i.e., MPE formula and test set method.

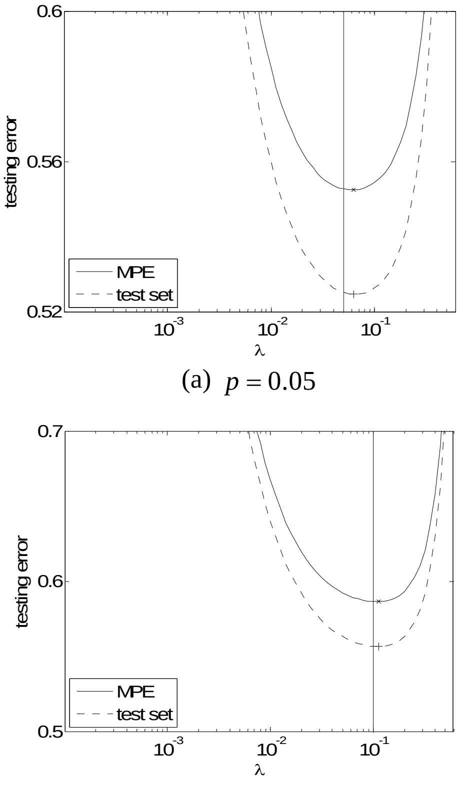
B. Multi-dimension function approximation

We consider the nonlinear autoregressive time series example as the multi-dimension function approximation problem [20]. Those function expression can be given as

$$\begin{aligned} y(i) = {} & \left(0.8 - 0.5\exp\left(-y^2(i-1)\right)\right) y(i-1) \\ & -\left(0.3 + 0.9\exp\left(-y^2(i-1)\right)\right) y(i-2), \\ & +0.1\sin\left(\pi y(i-1)\right) + n(i) \end{aligned} \tag{12}$$

where $n(i)$ is a mean-zero Gaussian random variable with variance 0.01. The RBF model is to predict $y(i)$ based on $y(i-1)$ and $y(i-2)$, i.e., $x(i) = \left[y(i-1), y(i-2)\right]$. In simulation, 500 training dataset and 500 testing dataset are generated based on the initial setting, $y(0) = y(-1) = 0$. Moreover, the Chen's LROLS method is applied to select important RBF centers [21]. The number of selected nodes is 40.

Fig.2 has shown the simulation results for the NAR time series prediction with two evaluation methods, i.e., MPE formula and the test set method, when $p = 0.05$ or $p = 0.1$. From the figures, we can find that the regularization parameters chosen by the MPE formula are very close to that by the test set method. For instance, in fig.2(a), when the fault rate is equal to 0.05, the selected regularization parameter for two methods are both about 0.063096. Furthermore, we have repeated the simulations for 50 times and found the range of optimal regularization parameter values chosen by the MPE formula is around from 0.056234 to 0.070795. In addition, we can obtain the similar results from fig.2(b).



(a) $p = 0.05$

(b) $p = 0.1$

Fig. 2 Optimal regularization parameter selection for nonlinear autoregressive time series prediction example with the different methods, i.e., MPE formula and test set method.

V. CONCLUSION

The paper derives a MPE formula to find the optimal regularization parameter for faulty neural networks. Simulation results have shown that our proposed formula method can quickly find the optimal regularization parameter, which is very close to the actual one by the conventional method, i.e., the test set method. It implies that ones can selection the optimal regularization parameter using our MPE formula instead of the test set method for faulty neural networks.

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