CS2026 ASSIGNMENT 5 (Due Date: April 10, 2026)

Instructions: You have to answer all of them. Put your answers in a MS WORD file, or other word processing file, and then submit the file to the course Gmail account.

Question 1

Figure 1 shows a simple digital logic circuit which consists of two logic gates, namely $LG1$ and $LG2$. In the following questions, the types of logic gates will be given. Fill in the truth table the output values of Z.

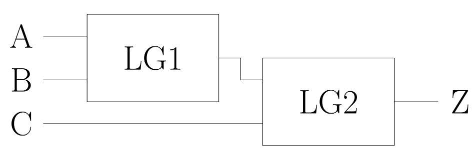

Figure 1: A three-input-one-output digital logic circuit which consists of two logic gates, $LG1$ and $LG2$.

A	B	C	Z
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

(a) $LG1$ is a NAND gate. $LG2$ is a NAND gate.

(b) $LG1$ is a NAND gate. $LG2$ is an AND gate.

(c) $LG1$ is an OR gate. $LG2$ is a NAND gate.

(d) $LG1$ is a NAND gate. $LG2$ is an OR gate.

(e) $LG1$ is an AND gate. $LG2$ is an XOR gate.

(f) $LG1$ is an XOR gate. $LG2$ is an AND gate.

Question 2

Figure 2 shows a simple processor with a single NAND gate, seven registers (R_1, R_2, R_3, R_4, R_A, R_B and R_Z), three simple switches (S_1, S_2 and S_3), eight two-input switches (S_{12}, S_{13}, S_{14}, S_{19}, S_{20}, S_{21}, S_{22}, S_{23}) and three control signals (A_1, A_2 and R/W).

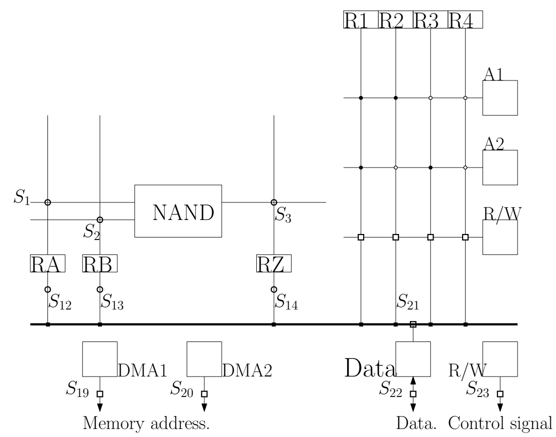

Figure 2: A simple processor with a single NAND gate, seven registers (R_1, R_2, R_3, R_4, R_A, R_B and R_Z), three simple switches (S_1, S_2 and S_3), eight two-input switches (S_{12}, S_{13}, S_{14}, S_{19}, S_{20}, S_{21}, S_{22}, S_{23}) and three control signals (A_1, A_2 and R/W).

It is assumed that R_1, R_2 and R_3 have already stored data. Design the micro-instructions for the following logical operations. Note that the instruction $MOVXY$ means copying the data in the register X to the register Y.

(a) (i) NAND R1 R2; (ii) MOV RZ R4. That is to say, $R4 = \overline{(R1R2)}$.

(b) The logical operation for Question 1(a). The values A, B and C have already been stored in R_1, R_2 and R_3. The output Z has to be stored in R_4.

(c) The logical operation for Question 1(f). The values A, B and C have already been stored in R_1, R_2 and R_3. The output Z has to be stored in R_4.

Question 3

In decimal (i.e. base 10) form, an non-negative integer X is represented as follows :

$$X_{10} = a_{n-1}a_{n-2}\cdots a_1a_0,$$

where a_k for $k = 0, \cdots, n-1$ is a symbol in the list $\{0, 1, 2, \ldots, 9\}$. The subscript 10 is used for highlighting that the number is in decimal form.

The value of X_{10} in decimal form can then be given by the following formulae.

$$X_{10} = a_{n-1} \times 10^{n-1} + a_{n-2} \times 10^{n-2} + \cdots + a_1 \times 10^1 + a_0 \times 10^0,$$

where The number n is the total number of digits.

For example, $X_{10} = 3219$ which is in decimal format. That is to say, $n = 4$ and

$$X_{10} = 3 \times 10^3 + 2 \times 10^2 + 1 \times 10^1 + 9 \times 10^0.$$

We get that

$$X_{10} = 3219_{10}.$$

For an n-bit positive binary (base 2) number, an non-negative integer X is represented as follows :

$$X_2 = b_{n-1}b_{n-2} \cdots b_1 b_0,$$

where b_k for $k = 0, \cdots, n-1$ is a symbol in the list $\{0, 1\}$. The value of X in decimal can be calculated by the following formulae.

$$X_{10} = b_{n-1} \times 2^{n-1} + b_{n-2} \times 2^{n-2} + \cdots + b_1 \times 2^1 + b_0 \times 2^0,$$

where the notation X_{10} is used for highlighting that the number is in decimal form. Again, n is the number of digits.

(a) Find X_{10} for $X_2 = 00000000$.

(b) Find X_{10} for $X_2 = 11111111$.

(c) Find X_{10} for $X_2 = 00001010$.

(d) Find X_{10} for $X_2 = 00011111$.

(e) Find X_{10} for $X_2 = 1111111111111111$.

(f) What is the minimum number (in decimal) of an non-negative binary number X_2 which is represented by 32 digits?

(g) What is the maximum number (in decimal) of an non-negative binary number X_2 which is represented by 32 digits?

(h) What is the second maximum number (in decimal) of an non-negative binary number X_2 which is represented by 32 digits?

Question 4

To handle a number with fractional part, we can specify the format of a binary number is represented as follows :

$$\begin{aligned} X_2 &= b_{n-1}b_{n-2} \cdots b_1 b_0 b_{-1} b_{-2} \cdots b_{-m} \\ &= b_{n-1}b_{n-2} \cdots b_1 b_0 . b_{-1} b_{-2} \cdots b_{-m}. \end{aligned}$$

Again, $b_k \in \{0, 1\}$ for $k = -m, \cdots, n$. The number in decimal can be obtained by the following formula.

$$X_{10} = b_{n-1} \times 2^{n-1} + \cdots + b_1 \times 2^1 + b_0 \times 2^0 + b_{-1} \times 2^{-1} + \cdots + b_{-m} \times 2^{-m}.$$

For simplicity, I will state the format of a binary number in the following form.

$$X_2 = xxxxxx.yy$$

One should understand that $n = 6$ and $m = 2$ in the above representation format.

(a) Find X_{10} for $X_2 = 101010.11$.

(b) Find X_{10} for $X_2 = 111111.11$.

(c) Find X_{10} for $X_2 = 000011.11$.

(d) Find X_{10} for $X_2 = 000000.01$.

(e) What is the minimum number that can be represented by a binary number with format $xxxxxx.yy$?

(f) What is the second minimum number that can be represented by a binary number with format $xxxxxx.yy$?

(g) What is the maximum number that can be represented by a binary number with format $xxxxxx.yy$?

(h) What is the second maximum number that can be represented by a binary number with format $xxxxxx.yy$?